Non-monotonic magnetoresistance of two-dimensional electron systems in the ballistic regime

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We report experimental observations of a novel magnetoresistance (MR) behavior of two-dimensional electron systems in perpendicular magnetic field in the ballistic regime, for $k_BT\tau/\hbar > 1$. The MR grows with field and exhibits a maximum at fields $B > 1/\mu$, where μ is the electron mobility. As temperature increases the magnitude of the maximum grows and its position moves to higher fields. This effect is universal: it is observed in various Si- and GaAs- based two-dimensional electron systems. We compared our data with recent theory based on the Kohn anomaly modification in magnetic field, and found qualitative similarities and discrepancies.

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Two-dimensional (2D) degenerate electronic systems of high purity ($k_Fl \gg 1$) with isotropic energy spectrum are rather simple objects, which seem to be well understood. Within the classical kinetic theory,[1] the resistivity of such a system should not depend on perpendicular magnetic field for $\omega_c\tau < 1$ (where $\omega_c = eB/m^*$ is the cyclotron frequency, and τ – the transport time). However, a noticeable magnetoresistance (MR) is often seen in experiments with 2D systems; such MR is usually attributed to quantum corrections which are beyond the classical consideration. There are two types of quantum corrections to conductivity: (i) weak localization (WL), and (ii) electron-electron (e-e) interaction correction (for a review, see Ref.[2]). In the diffusive regime ($k_BT\tau/\hbar \ll 1$, $\tau/\tau_\varphi \ll 1$), both corrections give rise to magnetoresistance with an amplitude proportional to $\ln(T)$[2,3] whereas in the ballistic regime ($k_BT\tau/\hbar > 1$, $\tau/\tau_\varphi > 1$) the magnetoresistance should disappear[3]. These theoretical predictions for the MR have been verified in diffusive and diffusive-to-ballistic crossover regimes in recent experiments[4,5,6] with 2D systems. A conventional belief (that the quantum corrections to MR have to disappear at high temperatures) has made the MR in purely ballistic regime out of the scope of experimental interests. This theory prediction for the ballistic regime, however, was not verified thoroughly. In order to shed light on this issue, we measured MR in the ballistic regime with various simple isotropic 2D electron systems. Contrary to the common belief we have found that the MR in perpendicular fields does not vanish at $k_BT\tau/\hbar > 1$; instead, it manifests a novel type of behavior: *the MR depends nonmonotonically on field and exhibits a maximum, whose position scales with temperature for all samples.*

In this paper, we report observation and systematic studies of the MR in the domain $k_Fl \gg 1$, $k_BT\tau/\hbar > 1$, where the MR should be missing. Experimentally, however, different Si-MOS structures, GaAs/AlGaAs heterostructures and GaAs-based quantum wells were found to show a nonmonotonic MR. Our results provide an evidence for a universal origin of the effect. We compared our data with a recent theory[7] of e-e interaction correction that employs modification of the Kohn anomaly by magnetic field and did find some qualitative similarities.

We used two Si-MOS samples (Si4, Si13 with peak mobilities 1-2 m^2/Vs) and GaAs-AlGaAs heterostructure 28, GaAs24 (mobility 24 m^2/Vs)[8], and gated quantum well structures AlGaAs-GaAs-AlGaAs (1520) and GaAs-InGaAs-GaAs (3513)[4]. All samples were patterned as Hall bars. Density of electrons in gated samples was varied in situ. The relevant parameters of the samples, densities n (in units of 10^{12}cm^{-2}), and mobilities μ (m^2/Vs), are summarized in the following table:

Si-samples	n	μ	GaAs-samples	n	μ
Si4	1.3	1.02	3513	1	2.2
Si4	1.7	1	28	0.35	24
Si4	2.35	0.96	24	0.4	21
Si4	3.4	0.93	1520	1.6	1.6
Si13	0.6	2.4	1520	1.4	1.5
Si13	0.7	2.3	1520	1	0.95
Si13	1	2.1	1520	0.8	0.8

Samples were inserted into a cryostat with a superconducting magnet; the field direction was always perpendicular to the 2D sample plane. Temperature was varied in the range 1.3-60 K. Both components of the resistivity tensor were measured simultaneously using the standard four-terminal technique with either SR-830 lock-in amplifier (samples Si4, Si13, 28, 24), or using rectangular current modulation (samples 1520,3513). Both, harmonic and rectangular modulation was made at frequencies 12-33 Hz. Current was chosen an order of 1 μA, to ensure the absence of electron overheating.

In order to exclude an admixture of the off-diagonal component of the resistivity, we swept magnetic field from $-B$ to B, and then symmetrized our data. Such a symmetrization is necessary for reliable measurements of corrections to the resistivity whose relative variations

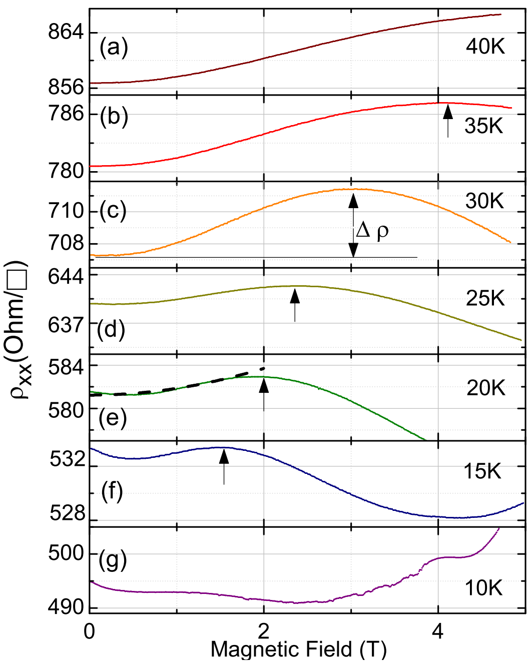

FIG. 1: (Color online)Magnetoresistance for sample Si4 at different temperatures. Electron density $n = 1.72 \cdot 10^{12}$ cm^{-2}. Up-arrows mark positions of the ρ_{xx} maxima. $\Delta\rho$ designates the magnitude of the MR. Dashed curve on the panel e shows fitting according to Eq. (2) with $\lambda^2 = 0.2$. $\hbar/k_B\tau \approx 8$K.

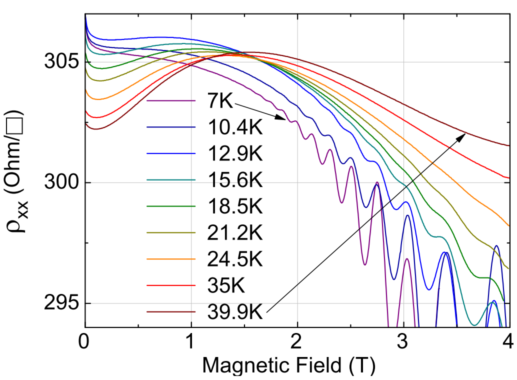

FIG. 2: (Color online)Magnetoresistance for sample 1520 at different temperatures. Electron density $n = 1.4 \cdot 10^{12}$ cm^{-2}. Temperature values are indicated in the figure. $\hbar/k_B\tau = 13.5$K.

might be less than 1%.

Electron density values quoted in the paper were determined from the slope of the Hall resistance versus B as well as from the period of Shubnikov - de Haas oscillations at low temperatures. Both results agreed with each other within 2%. The highest temperature in our experiments was chosen not to exceed 60 K in order the carrier density to remain constant and to avoid a bypassing bulk conductivity.

Examples of our MR curves, obtained at different temperatures for samples Si-4 and 1520 at fixed electron densities are shown in Figs. 1 and 2, respectively. As magnetic field is increased from zero, at first, ρ_{xx} sharply falls due to weak localization suppression. Then ρ_{xx} starts growing and reaches a maximum at $B^{\max}$ field (indicated by the arrows in Fig. 1). After passing the maximum ρ_{xx} decreases; in higher fields, $|B| > 1.5B^{\max}$, MR can become either positive or negative depending on the sample, temperature, electron density, etc. At the lowest temperatures, Shubnikov - de Haas oscillations are seen in high fields, on top of the smooth MR.

This unexpected nonmonotonic magnetoresistance is the main subject of the current paper. We stress that this effect (i.e., nonmonotonic MR) is universal. The point is that in different samples and at various electron densities it has similar features: (i) MR is small (its typical magnitude is less than 1%), (ii) the nonmonotonic MR is observed only for $T \geq 1.3\hbar/k_B\tau$[9], (iii) the MR maximum grows in magnitude and moves to higher magnetic fields as temperature increases (the position of the maximum exceeds $\omega_c\tau > 1$ and is roughly proportional to T).

Comparing the data from Figs. (1) and Fig. (2) for Si-MOSFET and GaAs QW-samples with similar mobilities and densities, we see that the MR takes a maximum at similar temperatures and magnetic fields, and at similar $\omega_c\tau$ values. This result indicates that the MR has an orbital rather than spin origin because the Zeeman energies $g^*\mu_B$ differ by a factor of 5 for these two different material systems. Also, this effect has nothing to do with WL and e-e-interaction diffusive corrections[2] because it survives at such high temperatures as $k_BT\tau/\hbar \approx 20$ for samples 28 and 24 at $T = 20$K.

Searching for possible semiclassical mechanisms, we have to note that most of the theoretical models for the case of short-range scatterers[10] predict a *negative, monotonic and temperature independent* magnetoresistance, due to the memory effects[11]. A positive, though T-independent, magnetoresistance was predicted in Ref.[12], due to non-markovian scattering. The latter type of MR was experimentally observed in very clean samples and for classically large magnetic fields[13], $\omega_c\tau \gg 1$. Therefore, we conclude that the aforecited semiclassical mechanisms can't explain the nonmonotonic MR observed in our experiments.

Recently, Sedrakyan and Raikh[7] suggested a new MR mechanism, which causes a maximum of resistivity in not-too-strong magnetic fields $\omega_c\tau \sim 1$ in the ballistic regime ($k_BT\tau/\hbar > 1$). This new mechanism seems to give the best starting point for comparison with our measurements. The MR in Ref.[7] originates from the e-e interaction correction to conductivity. According to Ref.[14], e-e interaction corrections to conductivity arise from scattering of electrons on Fridel's oscillations of electron density around impurities. Fridel's oscillations are a manifestation of the Kohn $2k_F$ anomaly in screening. Magnetic field applied perpendicular to the 2D plane modifies the electron spectrum and the Kohn anomaly; hence, the

field affects screening and electron scattering. In Ref. [7] this point was taken into account and shown to give rise to the second-order correction in the ballistic regime (see Eq. (5) from Ref.[7]):

$$\frac{\delta\sigma_{xx}}{\sigma_{xx}} = 4\lambda^2 \left(\frac{\pi k_B T}{E_F}\right)^{3/2} F_2 \left(\frac{\omega_c E_F^{1/2}}{2\pi^{3/2}(k_B T)^{3/2}}\right), \quad (1)$$

where $\lambda = 1 + 3F_0^\sigma/(1 + F_0^\sigma)$ is the interaction parameter[15].

Several predictions can be made based on this equation: (1) the correction to resistivity in small fields is always positive, (2) $(\delta\sigma_{xx}/\sigma_{xx})\cdot(E_F/T)^{3/2}$ is a universal function of $\omega_c E_F^{1/2}/T^{3/2}$ for a given interaction strength λ, and (3) MR has a maximum at $\omega_c\tau \approx 1/\sqrt{3}$.

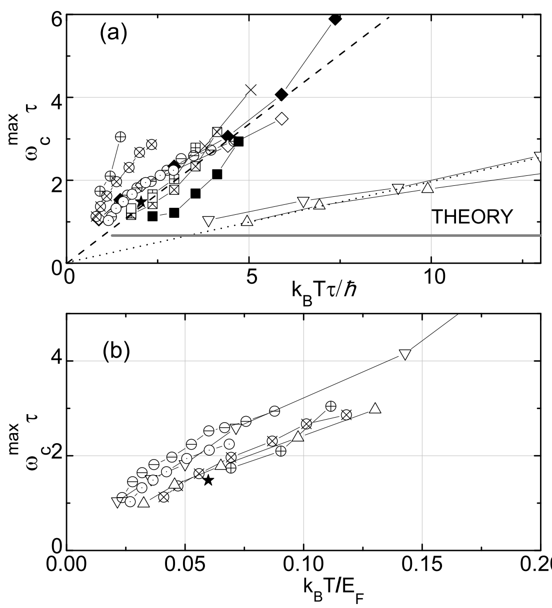

FIG. 3: (a) $\omega_c^{\max}\tau$ value versus dimensionless temperature $k_B T\tau/\hbar$ for all samples. Electron densities (in units of 10^{12} cm^{-2}) are ◇ - $n = 0.6$ (Si13);◆ - $n = 0.7$ (Si13);× - $n = 1$ (Si13); ■ - $n = 1.3$ (Si4); ⊠ - $n = 1.7$ (Si4); ⊞ - $n = 2.35$ (Si4);□ - $n = 3.4$ (Si4); ⊙ - $n = 1.4$ (1520); ⊗ - $n = 1$ (1520); ⊖ - $n = 1.6$ (1520); ⊕ - $n = 0.8$ (1520);⋆ - $n = 1$ (1520); ▽ - $n = 0.35$ (28); △ - $n = 0.4$ (24). Dashed line corresponds to $\hbar\omega_c^{\max} = 0.7k_B T$. Dotted line is $\hbar\omega_c^{\max} = 0.2k_B T$. Horizontal thick line is the theoretical prediction (see in the text). (b) The same data for GaAs-based samples solely scale in coordinates $\omega_c^{\max}\tau$ versus $k_B T/E_F$.

By comparing these theoretical predictions with our data, we find that prediction (1) is always fulfilled after subtraction of the weak localization. As for prediction (2), the $\rho_{xx}(B)$-data for different temperatures and over the whole range of magnetic fields do not scale as the theory predicts. Furthermore, the position of the MR maximum in our data is temperature dependent and corresponds to $\omega_c\tau \approx 1 - 3$, contrary to prediction (3). Moreover, the magnitude of the MR falls as temperature raises in the theory, whereas *in our experiment it grows with temperature*. Evidently, there is no complete agreement between the theory[7] and our experiment. We note finally, that the magnitude of the MR maximum $\Delta\rho$ seems do not scale with any dimensionless combination of $k_B T$, $\hbar/\tau$, $\hbar\omega_c^{\max}$ and E_F. This is also in contrast with the theory, where $\Delta\rho$ should be $\propto \hbar^2/[\tau^2 E_F^{0.5}(k_B T)^{1.5}]$.

According to Eq.1, the magnitude of the effect is proportional to the interaction constant λ^2. Therefore, one could estimate λ^2 value from the experimental data. In the theory, the maximum of MR inevitably results from $(1-\omega_c^2\tau^2)$ prefactor in resistivity tensor and should occur at $\omega_c\tau \approx 1/\sqrt{3}$. On the other hand, in the experiment the ρ_{xx} maximum is always observed at $\omega_c\tau > 1$, which indicates that this prefactor is weaker than in the theory. Therefore, for the order-of-magnitude comparison, we rewrite Eq. (1) for resistivity and omit the $[1-(\omega_c\tau)^2]$ prefactor:

$$\frac{\delta\rho_{xx}}{\rho_{xx}} = -4\lambda^2 \left(\frac{\pi k_B T}{E_F}\right)^{3/2} F_2 \left(\frac{\omega_c E_F^{1/2}}{2\pi^{3/2}(k_B T)^{3/2}}\right). \quad (2)$$

Example of the corresponding fitting with a single variable parameter λ^2 is shown in Fig. 1 e. The fit was performed in the limited range of magnetic fields $0.15\omega_c^{\max} < \omega_c < 0.65\omega_c^{\max}$, i.e. in the range of the applicability of Eq. (2) which ignores weak localization and the MR maximum. The λ^2 values obtained from the fit appeared to be temperature dependent i.e. grew monotonically from 0.1-0.4 to 1-3 as temperature was increased from $1.3\hbar/(k_B\tau)$ to maximal temperature. Surely this temperature dependence causes the lack of the scaling predicted by Eq. (1). Moreover, λ^2-values obtained from the fitting don't show a systematic dependence on carrier density and on material system.

On the other hand, the λ^2 value in our range of densities may be evaluated from the earlier measurements of $F_0^\sigma(n)$ parameter. The calculated λ values are T-independent and lie in the interval from 0.2 to 0.5 for GaAs-based structures[4] and from 1.5 to 5 for Si-based structures[16]. We conclude therefore that the observed MR disagrees qualitatively with the theory, though the theory predicts the MR of the right order of magnitude.

In Fig. 3 a, the position of the MR maximum of temperature. The $\omega_c^{\max}\tau$ value systematically exceeds the theoretical expectation $1/\sqrt{3}$ (horizontal thick line in Fig. 3 a) and approximately equals $0.7k_B T\tau/\hbar$ for most of the data (dashed curve in Fig. 3a). For samples with the highest mobility (24,28), the slope $\omega_c^{\max}\tau/(k_B T\tau/\hbar) \approx 0.2$ whereas for GaAs-based sample with the lowest mobility the slope exceeds 0.7. In order to take this fact into account we have applied another scaling, in coordinates versus $k_B T/E_F$ (see Fig. 3b). It is remarkable, that for GaAs-based 2D systems with mobilities and conductivities ranging by more than an order of magnitude, the $\omega_c^{\max}\tau$ data indeed scale reasonably, the result that might suggest a clue for understanding the effect.

The data for Si-based structures are not shown in Fig.3b because they fall out of the T/E_F scaling. In

order to understand the origin of the difference in scaling for Si- and GaAs- samples, we note that for GaAs-based samples in high fields, $B > B^{\max}$, the MR is always negative while for Si-based samples it can be either negative or positive, depending on particular sample and electron density. It means that some other mechanisms affect MR in Si-MOSFETs in strong perpendicular fields $B > B^{\max}$ and shift the MR maximum. It is also worthy of noting that in Si the discussed weak MR is observed at such high temperatures where metallic temperature dependence of the resistivity is strong and nonlinear with respect to T, and hence, the first order interaction corrections[14] are inapplicable.

We note also, that due to clear reasons the nonmonotonic MR in the ballistic regime $T \geq 1.3\hbar/k_B\tau$ was not oserved in the following cases: (i) Si-MOSFETs in the domain of strong interactions ($n < 6\cdot 10^{11}\mathrm{cm}^{-2}$) where the giant negative MR developes and dominates over other weak effects[17], (ii) Si-MOSFETs for such high temperatures where Fermi-gas is non-degenerate ($T/E_F \geq 0.5$), (iii) GaAs based samples at such high temperatures that the carrier density becomes B- and T-dependent.

Conclusions. In this paper we report experimental observation of the novel non-monotonic behavior of the magnetoresistance for 2D electron systems in perpendicular field. This MR is intrinsic to various 2D systems (Si-MOSFETs, GaAs and InGaAs quantum wells, and GaAs/AlGaAs-heterostructures) and occurs in the ballistic regime of high temperatures $T\tau > 1$. The MR is positive in low fields and reaches a maximum at $\omega_c\tau = 1-3$; the position of the maximum $\omega_c^{\max}$ scales linearly with temperature for all samples. We compare our data with recently suggested MR mechanism[7] and find some similarities: (i) the MR is always positive in low field, (ii) the MR exhibits a maximum in higher field and (iii) the MR magnitude is of the same order of magnitude as predicted. However, other features of our experimental data are in discrepancy with the theory Ref.[7]: (i) the MR maximum is achieved in fields which are noticeably higher than predicted, (ii) the position of the MR maximum linearly depends on temperature rather than remains constant, (iii) the magnitude of the effect increases with temperature rather than decreases, as predicted.

Some clue to understanding the effect may be provided by scaling of the MR maximum position $\omega_c^{\max}\tau$ versus T/T_F, which is empirically observed for various GaAs-samples in wide ranges of temperature, density and mobility. The observation of the nonmonotonic MR shows that the magnetotransport theory is still incomplete, at least for the ballistic regime, and requires further consideration.

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